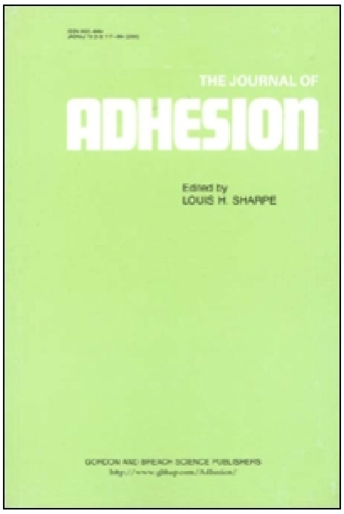

The Journal of Adhesion
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453635

The Stresses in an Adhesive Layer

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To cite this Article Harrison, N. L. and Harrison, W. J.(1972) 'The Stresses in an Adhesive Layer', The Journal of Adhesion, 3: 3, 195 — 212
To link to this Article: DOI: 10.1080/00218467208072193
URL: http://dx.doi.org/10.1080/00218467208072193

J. Adhesion, 1972, Vol. 3, pp. 195–212

Printed in Northern Ireland

The Stresses in an Adhesive Layer

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(*Received July* 5, 1971)

A simple method has been developed for calculating the stresses near the ends of a parallel-sided adhesive layer. The method can be applied to adhesive layers having aspect ratios of 10 or greater, and Poisson's ratios of 0.49 or less. For a layer subject to uniform boundary conditions of displacement at the adhering surfaces, the stress fields at distances greater than about five layer thicknesses from the free surfaces are uniform. The stress field throughout the layer is uniquely determined by the stresses in the uniform stress region. If the stress field is expressed by functions of reduced coordinates of position, obtained by dividing the cartesian coordinates by the layer thickness, these functions are for practical purposes independent of the aspect ratio or the thickness.

The method has been used to calculate shrinkage stresses, the stresses in a joint under tension perpendicular to the plane of the adhesive layer, and the stresses in a joint under shear. The features of the stress fields are described, and where necessary, shown in the form of graphs or contour plots.

1 INTRODUCTION

One of the factors which is likely to affect the performance of an adhesive joint is the distribution of stress within the adhesive layer. The stress can result from the load applied to the joint, or from strains whose equilibrium is maintained internally, due for example to shrinkage of the adhesive with respect to the adherends when the joint is formed. There are a number of published theories in which the stress distribution in adhesive joints is calculated[1,2,3]. In the theory of Goland and Reissner[2], the adherends are treated by beam theory, so that the results can only be expected to apply at distances from the ends of the joint comparable with or greater than the thickness of the adherends. The stress distributions calculated by Volkerson[1] and Cherry and Harrison[3] do not conform with the boundary condition that the shear stress must be zero at the free surfaces at the ends of the adhesive layer, and therefore cannot be valid near the ends. Wake[4] has attempted to calculate the shear stresses resulting from shrinkage of the adhesive, on the assumption that the adhesive is in uniform extension. Again, this is unlikely to apply near the free surfaces as the tensile stress perpendicular to a free surface must be zero.

All the problems treated by these authors are within the scope of the finite element method of stress analysis[5], and there is no special difficulty in obtaining stress values as close to the free surfaces of the adhesive as desired. In order that useful results can be obtained by this method, it is only necessary that the problems should be formulated in such a way that the results of a limited number of calculations can be applied generally. The problem treated in this paper is that of a layer of linearly elastic, isotropic material subjected to a simple set of boundary displacements. The layer is illustrated in Figure 1.

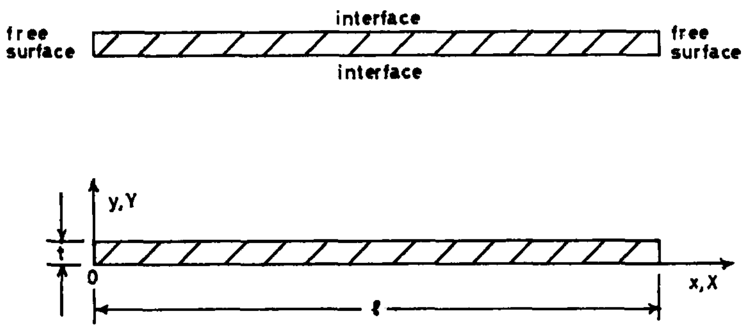

FIGURE 1 Diagrams showing dimensions of adhesive layer and coordinates of position.

The upper and lower surfaces (interfaces) are subject to displacements proportional to distance from any specified point in these surfaces, to uniform displacements towards or away from each other, and to displacements parallel to each other but in opposite directions. These situations correspond to an adhesive layer attached to rigid adherends, subject to shrinkage stresses, tension perpendicular to the layer, and shear.

FORMULATION OF THE PROBLEM

Consider an adhesive layer of length l and thickness t as shown in Figure 1. The layer extends indefinitely perpendicular to the plane of the paper, and it is assumed that there are no stress or strain variations in this direction. The usual coordinates (x, y) are reduced to non-dimensional form X, Y, where

$$X = \frac{x}{t}, \qquad Y = \frac{y}{t} \tag{1}$$

Two edges of the adhesive layer ($x = 0, x = l$) are free surfaces. On the other two surfaces, the boundary condition of displacement are as follows:

$$\left.\begin{array}{l} \text{On } y = 0, u = u_1(X)t, v = v_1(X)t \\ \text{On } y = t, u = u_2(X)t, v = v_2(X)t \end{array}\right\} \tag{2}$$

where u and v are displacements in the x and y directions respectively, and u_1 v_1, u_2 and v_2 are given functions of X.

From dimensional considerations (see Appendix), the stresses in such a layer, compared with other layers having the same aspect ratio (l/t), the same Poisson's ratio ν and boundary conditions as expressed by Eq. (2)

(i) are proportional to Young's modulus E;

(ii) are the same functions of X and Y.

We consider now a special case of the boundary conditions (2). Let the tensile strain parallel to the plane of the layer have a constant value A_1 at the interfaces. Let the upper interface be displaced a constant distance with respect to the lower interface, A_3t in the x direction, A_2t in the y direction. Under these circumstances

$$\left.\begin{array}{l} u_1(X) = A_1X, v_1(X) = 0 \\ u_2(X) = A_1X + A_3, v_2(X) = A_2 \end{array}\right\} \tag{3}$$

Let σ_x σ_y, τ_{xy} be the tensile and shear stresses throughout the layer. From the principle of linear superposition we can write

$$\left.\begin{array}{l} \sigma_x = E(B_{11}A_1 + B_{12}A_2 + B_{13}A_3) \\ \sigma_y = E(B_{21}A_1 + B_{22}A_2 + B_{23}A_3) \\ \tau_{xy} = E(B_{31}A_1 + B_{32}A_2 + B_{33}A_3) \end{array}\right\} \tag{4}$$

where B_{ij} are dimensionless quantities. From the statements (i) and (ii) above, B_{ij} are functions of X and Y but are independent of Young's modulus. They may be functions of the aspect ratio and Poisson's ratio.

The boundary conditions defined by Eqs. (3) correspond either to uniform strains at the adhering surfaces, or uniform displacements. On account of this, it seems likely that the stresses in the layer many thicknesses from the free surfaces will also be uniform. If this is the case, the strains in this region are related simply to A_1, A_2, and A_3:

$$\varepsilon_x = A_1,\ \varepsilon_y = A_2,\ \gamma_{xy} = A_3 \tag{5}$$

where ε_x and ε_y are the tensile strain parallel to the x and y directions respectively, and γ_{xy} is the shear strain referred to these axes. Stresses and strains in a region where the stress is uniform will be related by equations as follows (see for example Ref. 5, p.31).

$$\left.\begin{aligned} \sigma_x &= E_1(\varepsilon_x + \nu_1\varepsilon_y) \\ \sigma_y &= E_1(\nu_1\varepsilon_x + \varepsilon_y) \\ \tau_{xy} &= G\gamma_{xy} \end{aligned}\right\} \tag{6}$$

where

$$\left.\begin{aligned} E_1 &= \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \\ \nu_1 &= \frac{\nu}{1-\nu} \\ G &= \frac{E}{2(1+\nu)} \end{aligned}\right\} \tag{7}$$

If we use Eqs. (5) to re-write Eqs. (6) as

$$\left.\begin{aligned} \sigma_x &= E_1(A_1 + \nu_1 A_2) \\ \sigma_y &= E_1(\nu_1 A_1 + A_2) \\ \tau_{xy} &= G A_3 \end{aligned}\right\} \tag{8}$$

and compare Eqs. (8) with Eqs. (4), then in any region of uniform stress we will have

$$\left.\begin{aligned} B_{11} &= B_{22} = \frac{E_1}{E} \\ B_{12} &= B_{21} = \frac{\nu_1 E_1}{E} \\ B_{66} &= \frac{G}{E} \\ B_{13} &= B_{23} = B_{31} = B_{32} = 0 \end{aligned}\right\} \tag{9}$$

This suggests that a more appropriate form for the equations relating stresses to boundary conditions would be

$$\left.\begin{aligned} \sigma_x &= E_1(C_{11}A_1 + C_{12}A_2 + C_{13}A_3) \\ \sigma_y &= E_1(C_{21}A_1 + C_{22}A_2 + C_{23}A_3) \\ \tau_{xy} &= G(C_{31}A_1 + C_{32}A_2 + C_{33}A_3) \end{aligned}\right\} \quad (10)$$

where C_{ij} are functions of the reduced coordinates X and Y, the aspect ratio and Poisson's ratio. In a uniform stress region

$$\left.\begin{aligned} & C_{11} = C_{22} = C_{33} = 1 \\ & C_{12} = C_{21} = \nu_1 \\ & C_{13} = C_{23} = C_{31} = C_{32} = 0 \end{aligned}\right\} \quad (11)$$

It is unnecessary to assume the existence of a uniform stress region to derive Eqs. (10). They can be regarded at this stage simply as an alternative to Eqs. (4). Whether or not a uniform stress exists, and how close it would extend to the free surfaces, can be determined when the coefficients C_{ij} have been evaluated.

EVALUATION OF THE COEFFICIENTS C_{ij}

In this section, the stresses at any point in a layer subjected to three distinct sets of boundary conditions will be determined. These stresses and the values of A_1, A_2, and A_3 are substituted into Eqs. (10), yielding nine equations with nine unknowns C_{ij}. Solution of these nine equations gives the values of C_{ij} for the point in question.

There is one situation in which the stress and strain fields throughout the layer are particularly simple. A tensile stress σ_y uniform throughout the layer, with both σ_x and τ_{xy} zero, satisfies the conditions of equilibrium and continuity, and the boundary conditions at the free surfaces, and is therefore a possible stress configuration. For this condition with $\sigma_y = 1$, we have from Eqs. (8),

$$\left.\begin{aligned} 0 &= E_1(A_1 + \nu_1 A_2) \\ 1 &= E_1(\nu_1 A_1 + A_2) \\ 0 &= A_3 \end{aligned}\right\} \quad (12)$$

Hence we choose as the first set of boundary conditions

$$\left.\begin{aligned} A_1 &= \frac{-\nu_1}{E_1(1-\nu_1^2)} \\ A_2 &= \frac{1}{E_1(1-\nu_1^2)} \\ A_3 &= 0 \end{aligned}\right\} \tag{13}$$

resulting in a stress field

$$\sigma_x = 0, \quad \sigma_y = 1, \quad \tau_{xy} = 0 \tag{14}$$

for all values of X and Y. Substituting Eqs. (13) and (14) in (10) gives the first three equations.

$$\left.\begin{aligned} 0 &= -\nu_1 C_{11} + C_{12} \\ (1-\nu_1^2) &= -\nu_1 C_{21} + C_{22} \\ 0 &= -\nu_1 C_{31} + C_{32} \end{aligned}\right\} \tag{15}$$

For a layer of a particular aspect ratio and Poisson's ratio, two further stress fields corresponding to two sets of boundary conditions, necessary for solution of Eqs. (10), can be determined by finite element analysis. On account of the symmetry of the problem, it is necessary only to consider one quarter of the layer; the part chosen was the region $0 \leq Y \leq 0.5$ extending from $X = 0$ to one half of the layer length. Stresses were determined for layers having Poisson's ratios of 0.33 and 0.49. Incompressible materials ($\nu = 0.5$) cannot be treated easily by the finite element method, but the Poisson's ratio of 0.49 was used as an approximation to the condition. Young's modulus was taken as unity. The two sets of boundary conditions were chosen arbitrarily to be

$$\left.\begin{aligned} &A_1 = 0.01, A_2 = -0.1, A_3 = 0 \\ &A_1 = 0, \quad A_2 = 0, \quad A_3 = -1.0 \end{aligned}\right\} \tag{16}$$

While these correspond to very large strains, the method of computation is such that stresses are proportional to boundary displacements, no matter how large these are. The boundary conditions of Eqs. (16), together with the set given in Eqs. (15), provided the three sets necessary for solution of Eqs. (10).

An initial set of computations was carried out on layers of aspect ratios 10:1 and 100:1. The quadrant of the 10:1 layer with $X < 5$, $Y < 0.5$ was

TABLE I

Table of values of C_{ij}. $\nu = 0.33$

X	.025	.05	.05	.05	.05	.05	.15	.15	.15	.15	.15	0.5	0.5	0.5	0.5	0.5	1.5	1.5	1.5	1.5	1.5	5	5	5	5	5
Y	.025	0	.05	.1	.25	0.5	0	.05	.1	.25	0.5	0	.05	.1	.25	0.5	0	.05	.1	.25	0.5	0	.05	.1	.25	.05
C_{11}	.66	1.1	.34	.10	.06	0	.83	.64	.41	.16	0	.83	.77	.72	.54	.40	.98	.96	.94	.88	.91	1.0	1.0	.99	.99	.98
C_{12}	.33	.54	.17	.05	.03	0	.41	.32	.20	.08	0	.41	.38	.35	.27	.20	.48	.47	.46	.44	.45	.49	.49	.49	.49	.48
C_{13}	.17	.27	.08	.02	.01	.02	.14	.12	.08	.03	.01	.03	.03	.04	.03	.01	0	0	0	0	0	0	0	0	0	0
C_{21}	.79	.73	.66	.32	0	0	.18	.23	.24	.11	0	.20	.21	.23	.30	.33	.45	.44	.44	.44	.47	.49	.48	.49	.49	.49
C_{22}	1.1	1.1	1.1	.91	.70	.65	.84	.87	.87	.81	.75	.86	.86	.87	.90	.92	.98	.97	.97	.98	.99	1.0	1.0	1.0	1.0	1.0
C_{23}	.63	.55	.53	.44	.23	.05	.27	.28	.26	.16	.04	.04	.04	.04	.03	.01	0	0	0	0	0	0	0	0	0	0
C_{31}	2.2	3.1	2.1	1.0	.30	.22	2.4	2.4	2.0	.93	.16	1.3	1.3	1.2	.88	.11	.21	.26	.26	.26	.02	.01	.03	.03	.02	.01
C_{32}	1.1	1.5	1.0	.51	.15	.11	1.2	1.2	.97	.46	.08	.66	.63	.61	.43	.06	.10	.13	.13	.11	.01	.01	0.2	.01	.01	0
C_{33}	.59	.78	.61	.37	.21	.27	.70	.77	.76	.63	.50	.87	.89	.91	.97	.98	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

TABLE II

Table of values of C_{ij}. $\nu = 0.49$

X	.025	.05	.05	.05	.05	.05	.15	.15	.15	.15	.15	0.5	0.5	0.5	0.5	0.5	1.5	1.5	1.5	1.5	1.5	5	5	5	5	5
Y	.025	0	.05	.1	.25	0.5	0	.05	.1	.25	0.5	0	.05	.1	.25	0.5	0	.05	.1	.25	0.5	0	.05	.1	.25	0.5
C_{11}	.25	.32	.16	.10	.07	0	.22	.18	.14	.09	0	.31	.28	.26	.28	.18	.64	.57	.55	.49	.55	.81	.77	.76	.76	.73
C_{12}	.24	.30	.15	.09	.07	0	.22	.18	.14	.08	0	.30	.27	.25	.22	.17	.61	.55	.53	.47	.53	.78	.74	.74	.73	.71
C_{13}	.03	.04	.02	.01	0	.01	.02	.01	.01	03	.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{21}	.35	.26	.22	.16	.03	0	.16	.14	.11	.06	.03	.24	.22	.21	.22	.22	.59	.52	.51	.48	.55	.76	.72	.73	.73	.72
C_{22}	.41	.32	.29	.23	.11	0	.23	.21	.19	.13	.10	.31	.30	.28	.29	.29	.64	.58	.57	.53	.60	.81	.77	.77	.78	.77
C_{23}	.05	.04	.04	.03	.02	0	.02	.02	.02	.01	.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{31}	11	12	9.2	5.9	2.4	2.5	9.9	9.3	7.7	5.5	2.0	8.7	8.1	7.5	5.0	.89	5.2	5.3	4.8	3.2	.28	2.3	2.6	2.4	1.6	.20
C_{32}	11	11	8.9	5.6	2.2	2.3	9.5	8.9	7.4	5.3	1.9	8.4	7.8	7.2	4.9	.86	4.9	5.1	4.6	3.1	.27	2.2	2.5	2.3	1.5	.19
C_{23}	1.1	.97	.91	.58	.28	.52	.76	.86	.87	.78	.70	.81	.86	.90	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

divided into 190 triangular elements, with a concentration of small elements near the corner ($X = 0$, $Y = 0$) where the stresses were expected to vary most rapidly with position. The subdivision of the 100:1 layer was identical with that of the 10:1 layer in the region $X < 5$, $Y < 0.5$, but further elements were added in the region $X > 5$ to give a total of 315 elements. The stresses were determined by means of a computer program essentially the same as that published in Reference 5, but with provision for nodal averaging (see Reference 5). The coefficients C_{ij} were calculated from the nodal averages of the stresses.

Variation of the aspect ratio from 10:1 to 100:1 had only a small effect on the values of C_{ij}. For $\nu = 0.33$, the differences due to variation of the aspect ratio were less than 1% throughout. For $\nu = 0.49$, the effect was greater, with most differences of the order of 5% and some up to 10%. With the coefficients C_{31} and C_{32}, the percentage differences were in some places larger than 10%, but only where the values of the coefficients were insignificantly small.

Variation of C_{ij} with position (X, Y) was significant only within a few layer thicknesses of the free surfaces for both Poisson's ratios. The region of variation was very much smaller with $\nu = 0.33$ than with $\nu = 0.49$. Since C_{ij} do not vary except near the ends of the layer, the stress fields associated with surface displacements of the form given by Eqs. (3) are uniform throughout most of the layer. C_{ij} in the uniform stress region were found to have the values predicted in Eqs. (11).

A third mesh of triangular elements was constructed, corresponding to an aspect ratio of 20:1, containing three times the density of elements in the regions where significant stress variations were detected with the previous meshes. The portion of this mesh containing the corner is shown in Figure 2. C_{ij} were calculated as before using stresses determined from this mesh. For $\nu = 0.33$, these agreed well (within about 10% of their values, where these were of significant magnitude) except in the region $X < 0.03$. For $\nu = 0.49$, the differences were larger, typically of the order of 20% in the region where the stresses varied, but again with larger discrepancies close to the free surface. In all cases, the stresses well away from the free surface ($X > 3$) agreed very closely with those obtained previously. Certain of the stresses rose rapidly near the corner, and the maxima of these obtained with the fine mesh were as much as $2\frac{1}{2}$ times the values obtained previously. It is likely that these are in fact infinite at the corner.

On account of the similarity of the calculated values of C_{ij} corresponding to the aspect ratios of 10:1 and 100:1, it is reasonable to assume that these quantities are independent of aspect ratio provided this is greater than 10:1. Aspect ratios of less than 10:1 have not been treated, since adhesive layers of such low aspect ratio are unlikely to occur in practice. The values

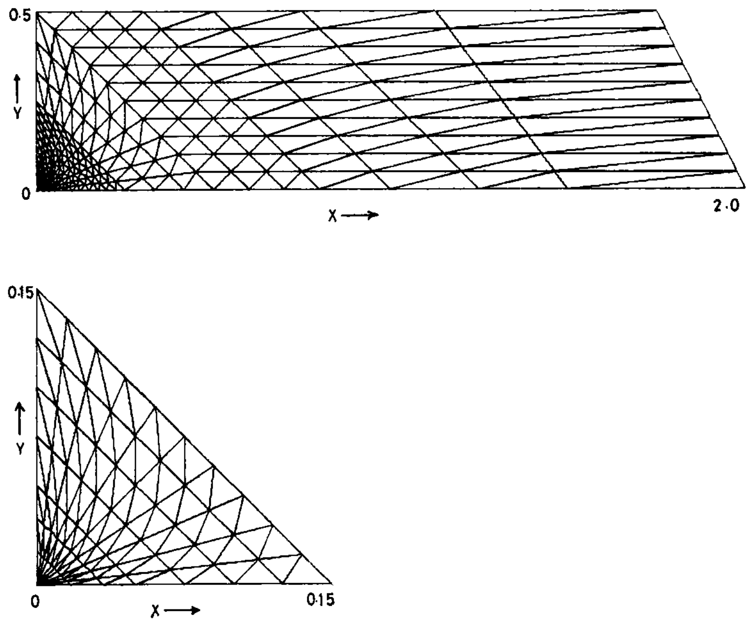

FIGURE 2 Portion of finite element mesh used for determination of C_{ij}.

of C_{ij} obtained with the fine mesh of 20:1 aspect ratio should be considerably more accurate than those obtained with the other meshes, and some of these have been tabulated in Tables 1 and 2. All numerical results presented subsequently were obtained from this mesh also.

APPLICATIONS

The loads on a joint are more easily determined in practice than the relative displacements of the interfaces. In sub-sections 1, 2 and 3 following, situations are considered in which the boundary conditions of the adhesive layer conform to Equations (3), and it is shown how A_1, A_2 and A_3 and hence the stress fields, can be calculated from applied loads. The adherends are assumed to be perfectly rigid. The stresses in the layer are due to shrinkage with respect to the adherends (Figure 3a), tension perpendicular to the plane of the layer, symmetrical about the centre of the layer, applied to the adherends (Figure 3b), and shear stresses applied to the adherends (Figure 3c).

For the cases in which the joint is subject to external loads, it will be assumed that there are no shrinkage stresses.

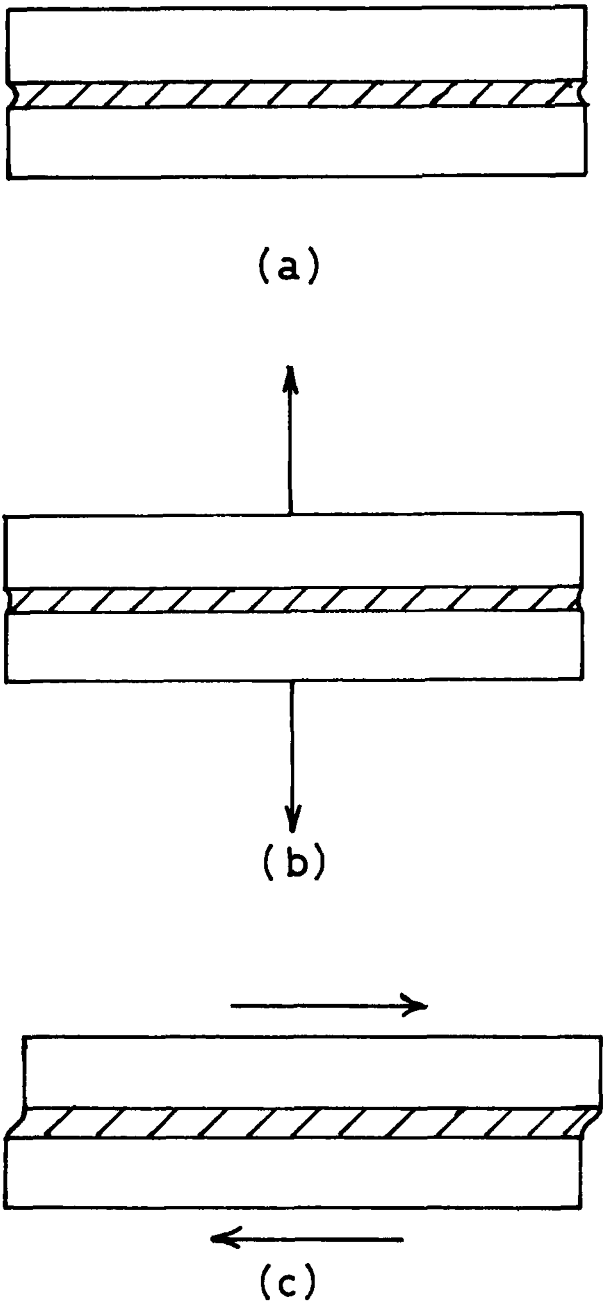

FIGURE 3 Types of loading for which stress fields are calculated.

1. Shrinkage Stresses (Figure 3a)

If an adhesive shrinks after it has bonded to a rigid surface, it retains a tensile strain at the surface equal to the linear contraction per unit length which it would undergo if it were free. This quantity will be referred to as the relative shrinkage. A_1 is equal to the relative shrinkage. If the adherends are free of externally applied loads, A_3 is zero by symmetry, and A_2 is such as to make the mean value of σ_y at the interfaces zero, i.e.

$$\int_o^l \sigma_y \, dx = 0 \tag{17}$$

An expression can be found for the integral in terms of A_1, A_2, A_3, and C_{ij} and Eq. (17) solved to give A_2. Alternatively, if the aspect ratio of the joint is large, A_2 can be calculated on the assumption that A_2 is such as to make σ_y zero in the uniform stress region; i.e. from Eqs (8)

$$A_2 = -\nu_1 A_1 \tag{18}$$

Shrinkage stresses were calculated using the two methods indicated for determining A_2, for an aspect ratio of 100:1. The differences were insignificant for both $\nu = 0.33$ and $\nu = 0.49$. The stress field was uniform throughout the layer, except within about five layer thicknesses from the ends. Near the ends, σ_y was compressive, while σ_x fell below its value in the uniform stress region. The maximum of τ_{xy} corresponding to any value of X occurred at or near an interface, and diminished with increasing distance from a free surface.

In Figure 4, the value of τ_{xy} at the interface is plotted as a function of X. τ_{xy} is proportional to the stress σ_x in the uniform stress region, and the curves shown in Figure 4 correspond to $\sigma_x = 1$. There is a much greater concentration of stress for $\nu = 0.33$ than for $\nu = 0.49$.

The tensile stress σ_x in the uniform stress region is easily calculated from Eqs. (10), (11) and (18).

$$\sigma_x = (1 - \nu_1^2) E_1 A_1$$

It depends only on the relative shrinkage and elastic constants. The shear stresses, relative to this tensile stress, depend only on the coordinates of position. They are virtually independent of the thickness of the joint and the aspect ratio. This is in accordance with the argument presented by Dukes and Bryant[6] and contrary to that of Wake[4].

2. Joint under tension (Figure 3b)

If the adherends of a joint are subjected to symmetrically placed tensile forces acting perpendicular to the plane of the adhesive layer, and if the

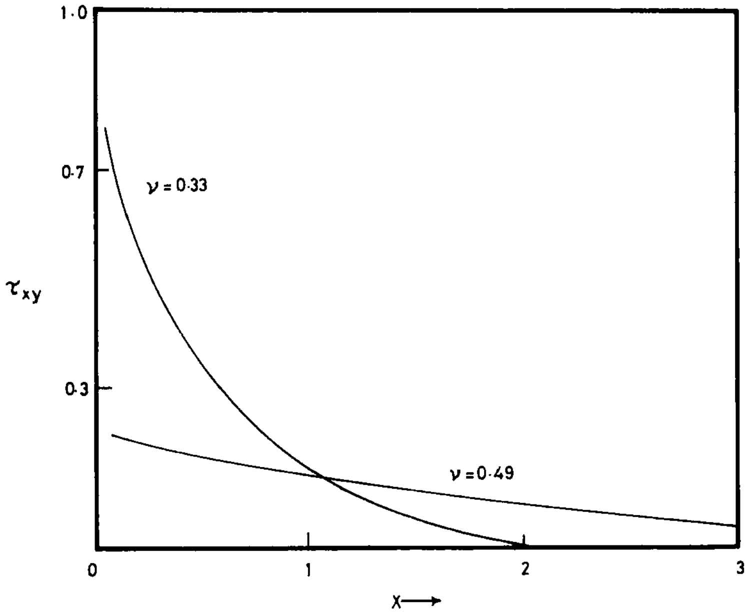

FIGURE 4 Interfacial shear stress resulting from shrinkage of the adhesive. σ_x in the uniform stress region = 1.0.

relative shrinkage is zero, A_1 and A_3 are zero. If the total force is $F_\perp$ and the total area of the joint A, the mean tensile stress σ_y at the interface is $F_\perp/A$. It can be assumed that the non-uniform stresses near the free surfaces have a negligible effect in determining the relative displacements of the adherends, and so σ_y in the uniform stress region is equal to $F_\perp/A$. From Equations (8) we then have

$$A_2 = \frac{F_\perp}{AE_1} \tag{19}$$

Calculation of the stress field from Eqs. (10) showed that the stress field in the regions where it is not uniform is similar in form to that arising from shrinkage. Both σ_y and σ_x are less than in the uniform stress region, while τ_{xy} has an appreciable value only near the free surfaces. The interfacial shear stresses corresponding to a mean σ_y of unity are plotted in Figure 5.

3. Joint under shear (Figure 3c)

With the loading situation shown in Figure 3c, any relative movement of the adherends perpendicular to the plane of the adhesive layer arises from the moment of the force acting on the upper adherend about the lower adherend.

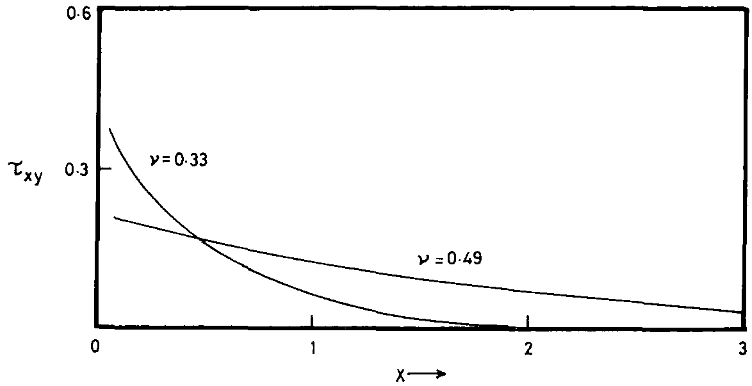

FIGURE 5 Interfacial shear stress resulting from tension perpendicular to the plane of the layer. σ_y in the uniform stress region = 1.0.

If the joint is long relative to its thickness, we can assume this movement has a negligible effect on the displacements of the adherends, and can take A_2 as zero. A_1 is zero since there is no relative shrinkage. We can assume that A_3 is determined by the shear stress in the uniform stress region, which is equal to $F_{\|}/A$ where $F_{\|}$ is the force parallel to the adhesive layer acting on one of the adherends. From Eqs. (8)

$$A_3 = F_{\|}/AG \tag{20}$$

Calculation of the stress field showed that throughout most of the joint, σ_x and σ_y are zero. However, close to the free surfaces they rise to values well above the uniform shear stress. The shear stress τ_{xy} is lower near the free surfaces than elsewhere. The interfacial tension σ_y is plotted in Figure 6. Stresses above the mean shear stress extend well away from the interface into the adhesive layer. Since the adhesive is isotropic it is desirable to express the stresses in the layer itself, rather than at the interface, in a manner independent of the system of co-ordinates. Accordingly, the principal stresses σ_1 and σ_2, and the maximum shear stress, τ, given by

$$\tau = \tfrac{1}{2}(\sigma_1 - \sigma_2) \tag{21}$$

were calculated.

The values obtained for the maximum shear stress rose above unity, for unit shear stress in the uniform stress region, only in the small region $X < .03$, $Y < .03$, and in that region were less than half the greater of the two principal stresses (i.e. σ_1 and σ_2 were of the same sign). The greater of the two principal stresses rose appreciably above the mean shear stress over a larger portion of the layer. The magnitude and extent of this enhanced stress are shown in

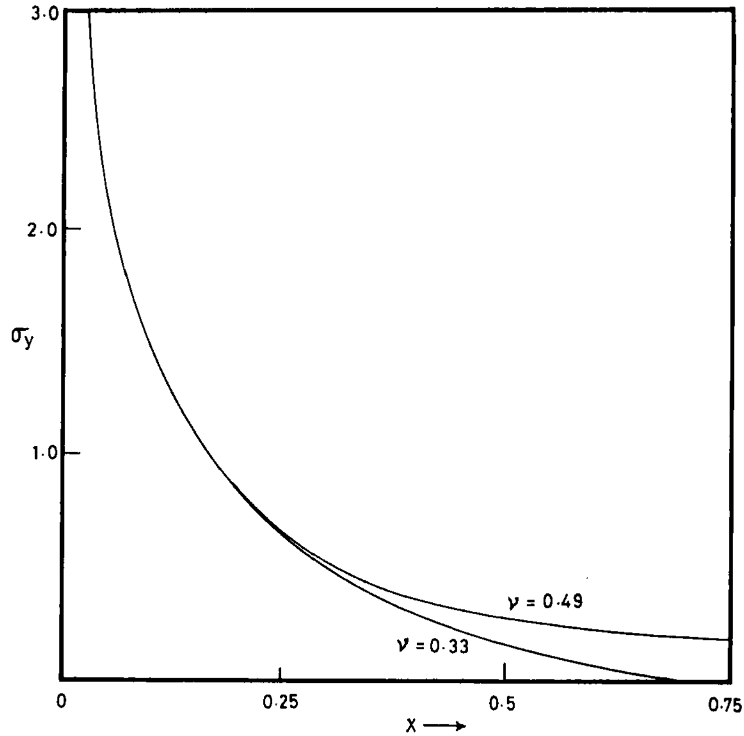

FIGURE 6 Interfacial tension for joint under shear. τ_{xy} in the uniform stress region = 1.0.

the contour plots, Figures 7 and 8. These show the part of the layer with $0 < X < 0.75$, $0 < Y < 0.50$. The plots for other parts of the layer close to the free surfaces would be mirror images of the one shown. For a positive shear stress applied to the adherends, the greater principal stress in the region shown is tensile; this is also the case in the diagonally opposite corner of the layer. The greater principal stress in the other corners is compressive.

For any value of $F_{||}/A$, the magnitudes of the non-uniform stresses are independent of the layer thickness, but the volume of adhesive in which magnified stress occurs increases with increasing thickness of the adhesive layer. It has been found experimentally that the strengths of joints diminish with increasing thickness of the adhesive layer (e.g. References 6, 7). If failure of the joint were governed by the stresses in uniform stress regions, the dependence of strength on thickness could result from the greater volume of adhesive under stress, according to flaw theories of failure[6]. The strength of the joint would be a function of the total volume of adhesive under stress.

However, if failure is governed by the concentrated stresses near the free surfaces, there would be no dependence on the length of the layer, but only on its thickness.

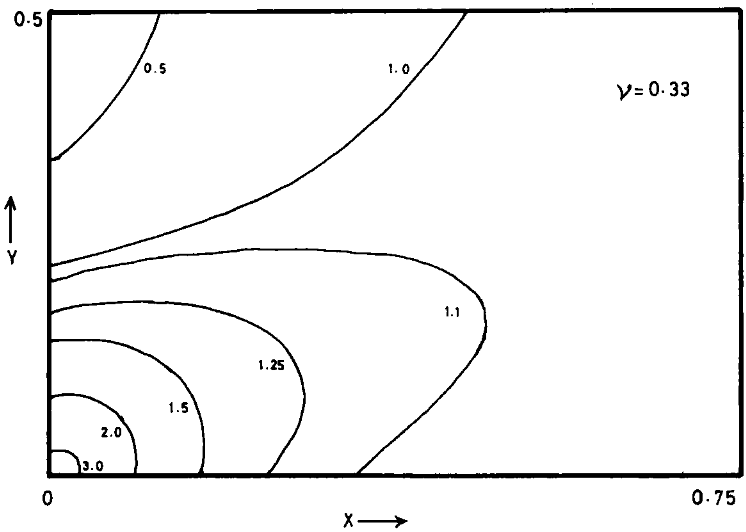

FIGURE 7 Contour plot of greater principal stress in joint under shear, for $\nu = 0.33$.

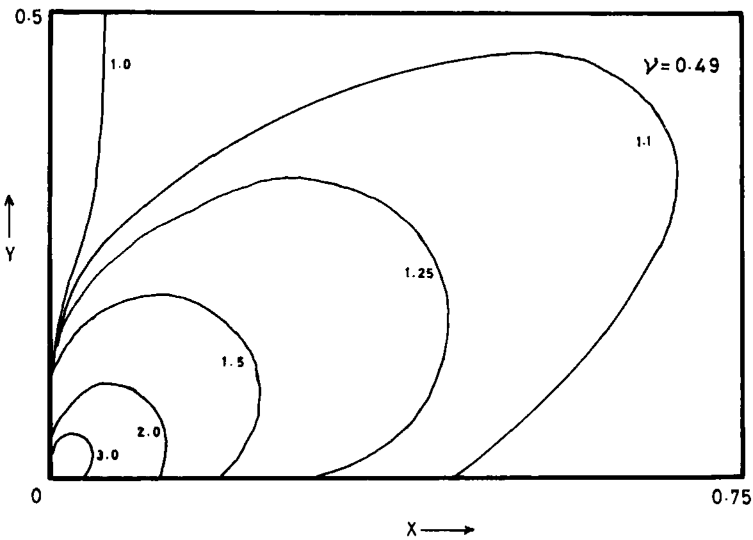

FIGURE 8 Contour plot of greater principal stress in joint under shear, for $\nu = 0.49$.

4. Furthern Applications

As pointed out in the introduction, there are a number of published methods for calculating the stresses in adhesive joints of various configurations which are applicable to regions in the adhesive layer well away from the free surfaces[1,2,3]. In the usual practical situations in which the adherends are thick compared with the adhesive layer, or of higher modulus, the stress fields calculated by these methods vary only slightly over many layer thicknesses. Since the rate of variation of stress is slight, the relative displacements of the surfaces of the adherends over the entire regions to which these methods are applicable can be calculated by solving Eqs. (8). Furthermore, unless the adherends are particularly flexible, the displacements a few layer thicknesses from the ends of the joint can be assumed to be the same up to the ends. The stress field up to the free surfaces of the adhesive layer, except within a small fraction of the layer thickness from the corners can then be calculated from the known values of A_1, A_2 and A_3, the coefficients given in Tables 1 and 2 and Eqs. (10).

CONCLUSIONS

The general conclusions which can be drawn from this work apply to adhesive layers having aspect ratios of 10 or greater, and Poisson's ratios of 0.49 or less. For a layer subject to uniform boundary conditions of displacement at the adhering surfaces, the stress fields at distances greater than about five layer thicknesses from the free surfaces are uniform. The stress field throughout the layer is determined uniquely by the stresses in the uniform stress region. If the stress field is expressed by functions of reduced coordinates of position, obtained by dividing the cartesian coordinates by the layer thicknesses, these functions are independent of thickness and for practical purposes are independent of the aspect ratio.

If the adhesive has shrunk relative to the adherends, or if the joint is subject to tension perpendicular to the plane of the adhesive layer, there is a concentration of shear stress at the interfaces near the ends of the joint. In a joint subject to an applied shear stress, there is a concentration of tensile stresses in the adhesive near its free surfaces, and the stresses in this region rise to several times the mean applied stress.

The data necessary for calculating stresses in adhesive layers subject to uniform boundary displacements have been tabulated. These data can be used to calculate the stresses near the free surfaces of adhesive layers in which the boundary displacements are not strictly uniform, but vary only slightly over lengths several times the layer thickness.

Acknowledgment

The authors are grateful to Dr. B. W. Cherry, at whose suggestion this work was undertaken.

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APPENDIX

Consider two adhesive layers, one of thickness t and length l, and the other of thickness unity and length l/t. Both layers have the same Poisson's ratio. Layer 1 has Young's modulus E and layer 2 has Young's modulus unity.

The boundary conditions for layer 1 are

$$\sigma_x = \tau_{xy} = 0 \qquad \text{at the free surfaces}$$

$$u = u_1\left(\frac{x}{t}\right)t, \quad v = v_1\left(\frac{x}{t}\right)t \text{ on } y = 0$$

$$u = u_2\left(\frac{x}{t}\right)t, \quad v = v_2\left(\frac{x}{t}\right)t \text{ on } y = t$$

The boundary conditions for layer 2 are

$$\sigma_x = \tau_{xy} = 0 \qquad \text{at the free surfaces}$$

$$u = u_1(x), \quad v = v_1(x) \text{ on } y = 0$$

$$u = u_2(x), \quad v = v_2(x) \text{ on } y = 1$$

In layer 1, we introduce new coordinates

$$X = \frac{x}{t}, \qquad Y = \frac{y}{t}$$

new units of displacement,

$$U = \frac{u}{t}, \qquad V = \frac{u}{t}$$

and new units of stress

$$S_x = \frac{\sigma_x}{E}, \quad S_y = \frac{\sigma_y}{E}, \quad T_{xy} = \frac{\tau_{xy}}{E}$$

In these new units, the boundary conditions for layer 1 become

$$S_x = T_{xy} = 0 \qquad \text{at the free surfaces}$$

$$U = u_1(X), \quad V = v_1(X) \text{ on } Y = 0$$

$$U = u_2(X), \quad V = v_2(X) \text{ on } Y = 1$$

which are the same as those of layer 2. Similarly, the stress-strain relations of layer 1 in the new units are the same as those of layer 2 in the original units. Since the equilibrium and compatibility equations are valid in any units, it follows that the displacements U, V, in layer 1 equal the displacements u, v in layer 2, while S_x, S_y, T_{xy} in layer 1 equal σ_x, σ_y, τ_{xy} in layer 2.